9709_11_Summer_2020_Q1

Solution

To find the first term and the common difference of the **arithmetic progression** (AP), we utilize the formula for the sum of the first n terms, S_n:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a is the **first term** and d is the **common difference**.

1. Formulate the first equation from the sum of the first nine terms The sum of the first nine terms (S_9) is given as 117.

$$S_9 = \frac{9}{2}[2a + (9-1)d] = 117$$

$$\frac{9}{2}[2a + 8d] = 117$$
$$9(a + 4d) = 117$$
$$a + 4d = 13 \quad \text{... (Eq. 1)}$$

2. Formulate the second equation from the sum of the next four terms The sum of the "next four terms" refers to the terms from the 10^{th} to the 13^{th}. This can be expressed as the difference between the sum of the first 13 terms (S_{13}) and the sum of the first 9 terms (S_9).

$$S_{13} - S_9 = 91$$
$$S_{13} = 91 + S_9$$
$$S_{13} = 91 + 117 = 208$$

Now, apply the sum formula for $n = 13$:

$$S_{13} = \frac{13}{2}[2a + (13-1)d] = 208$$

$$\frac{13}{2}[2a + 12d] = 208$$
$$13(a + 6d) = 208$$
$$a + 6d = 16 \quad \text{... (Eq. 2)}$$

3. Solve the system of linear equations Subtract (Eq. 1) from (Eq. 2) to eliminate a:

$$(a + 6d) - (a + 4d) = 16 - 13$$
$$2d = 3$$
$$d = 1.5$$

Substitute $d = 1.5$ back into (Eq. 1) to find a:

$$a + 4(1.5) = 13$$
$$a + 6 = 13$$
$$a = 7$$

The first term of the progression is 7 and the common difference is 1.5.

$\boxed{a = 7, d = 1.5}$

9709_11_Summer_2020_Q2

Solution

To find the value of the positive constant k, we analyze the **binomial expansion** of the two given expressions and identify the terms containing $1/x$ (or x^{-1}).

1. Expansion of the first term The first part of the expression is $\left(kx + \frac{1}{x}\right)^5$. Using the **Binomial Theorem**, the general term T_{r+1} is given by:

$$T_{r+1} = \binom{5}{r}(kx)^{5-r}\left(\frac{1}{x}\right)^r = \binom{5}{r}k^{5-r}x^{5-2r}$$

We require the coefficient of x^{-1}, so we set the exponent of x to -1:

$$\begin{aligned} 5 - 2r &= -1 \\ 2r &= 6 \\ r &= 3 \end{aligned}$$

The coefficient from this expansion is:

$$C_1 = \binom{5}{3}k^{5-3} = 10k^2$$

2. Expansion of the second term The second part of the expression is $\left(1 - \frac{2}{x}\right)^8$. The general term is:

$$T_{s+1} = \binom{8}{s}(1)^{8-s}\left(-\frac{2}{x}\right)^s = \binom{8}{s}(-2)^s x^{-s}$$

We require the coefficient of x^{-1}, so we set $s = 1$. The coefficient from this expansion is:

$$C_2 = \binom{8}{1}(-2)^1 = 8 \cdot (-2) = -16$$

3. Solving for k The total coefficient of $1/x$ in the sum of the two expansions is given as 74. Therefore:

$$\begin{aligned} C_1 + C_2 &= 74 \\ 10k^2 - 16 &= 74 \\ 10k^2 &= 90 \\ k^2 &= 9 \\ k &= \pm 3 \end{aligned}$$

Since the problem specifies that k is a **positive constant**, we take the positive root.

$\boxed{k = 3}$

9709_11_Summer_2020_Q3

Solution

The problem describes a scenario where the selling price of a diamond necklace increases by a fixed percentage each year. This is a classic application of a **geometric progression** or **compound interest**.

1. Expression for the selling price n years later

The initial price in the year 2000 is $P_0 = \$36,000$. The annual increase is 5%, which corresponds to a growth factor of $r = 1 + 0.05 = 1.05$.

- Let P_n be the selling price n years after 2000.
- The price follows the formula for **exponential growth**;:

$$P_n = P_0 \times (1.05)^n$$

- Substituting the initial value:

$$P_n = 36000 \times (1.05)^n$$

2. Selling price in 2008

To find the price in 2008, we calculate the number of years since 2000:

$$n = 2008 - 2000 = 8$$

- Substitute $n = 8$ into the expression:

$$\begin{aligned} P_8 &= 36000 \times (1.05)^8 \\ &\approx 36000 \times 1.47745544 \\ &\approx 53188.3959 \end{aligned}$$

Rounding to two decimal places for currency:

$$\boxed{P_n = 36000 \times (1.05)^n;\ \text{Price in 2008 } = \$53,188.40}$$

3. Total amount of money obtained in the ten-year period

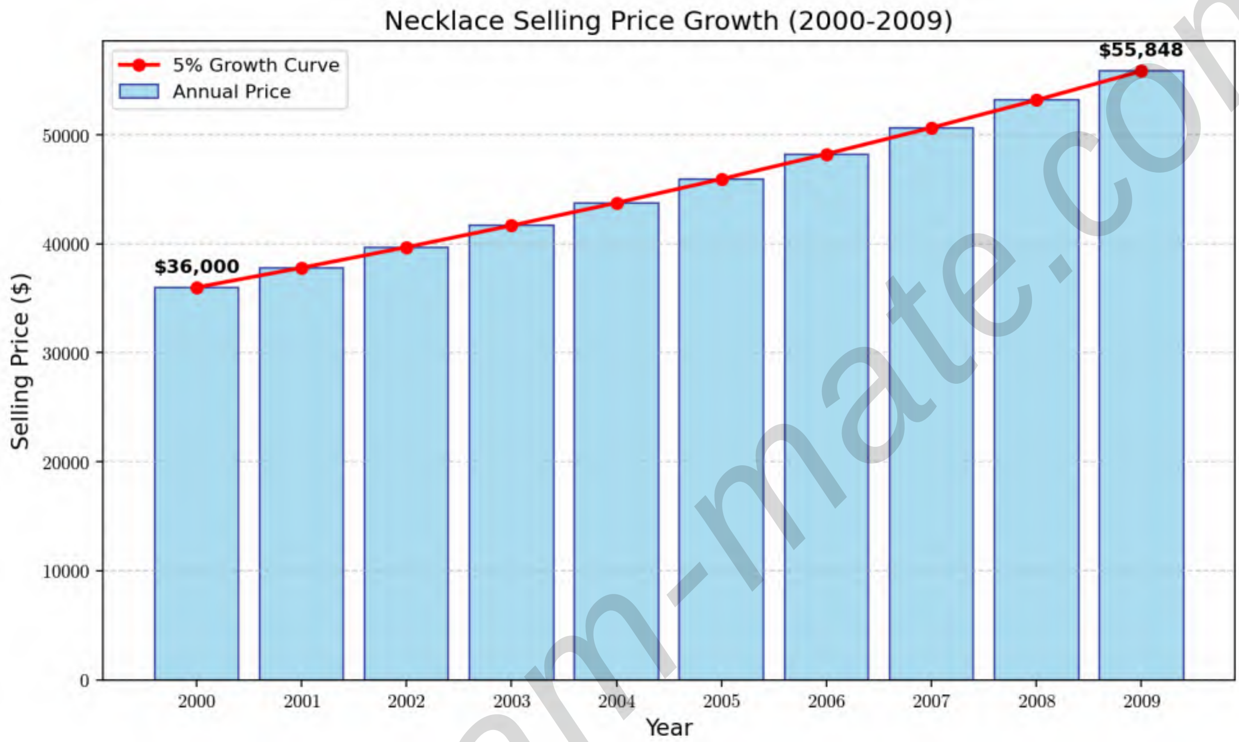
The company sells one necklace each year for ten years, starting from the year 2000 ($n = 0$) to the year 2009 ($n = 9$). The total amount is the sum of a **geometric series** with $a = 36000$, $r = 1.05$, and $k = 10$ terms.

- The formula for the sum of the first k terms of a geometric series is:

$$S_k = \frac{a(r^k - 1)}{r - 1}$$

- Substituting the values:

$$\begin{aligned}S_{10} &= \frac{36000((1.05)^{10}-1)}{1.05-1} \\ &= \frac{36000(1.6288946-1)}{0.05} \\ &= \frac{36000(0.6288946)}{0.05} \\ &= 720000 \times 0.6288946 \\ &\approx 452804.13\end{aligned}$$



Rounding to the nearest cent:

$$\boxed{\$452,804.13}$$

9709_11_Summer_2020_Q4

Solution

The problem asks for an analysis of the trigonometric function $f(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ over the interval $0 \leq x \leq \pi$.

1. Determine the range of f The **range** of a trigonometric function of the form $y = A\cos(Bx) + C$ is determined by its **amplitude** $|A|$ and its **vertical shift** C.

- For $f(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$, the amplitude is $A = \frac{3}{2}$ and the vertical shift is $C = \frac{1}{2}$.
- The maximum value of $\cos 2x$ is 1, and the minimum value is -1.
- The maximum value of $f(x)$ is:

$$f_{\max} = \frac{3}{2}(1) + \frac{1}{2} = 2$$

- The minimum value of $f(x)$ is:

$$f_{\min} = \frac{3}{2}(-1) + \frac{1}{2} = -1$$

  The range of f is $-1 \leq f(x) \leq 2$.

2. Determine the constant k and describe the transformation The function $g(x)$ is defined as $g(x) = f(x) + k$, where $k > 0$. This represents a **vertical translation** of the graph of $f(x)$ upwards by k units.

- It is given that the x-axis is a **tangent** to the curve $y = g(x)$. Since k is positive, the graph is shifted upwards. For the x-axis ($y = 0$) to be a tangent to the curve, the minimum point of $g(x)$ must lie on the x-axis.
- The minimum value of $g(x)$ is:

$$g_{\min} = f_{\min} + k = -1 + k$$

- Setting the minimum value to zero:

$$-1 + k = 0 \Longrightarrow k = 1$$

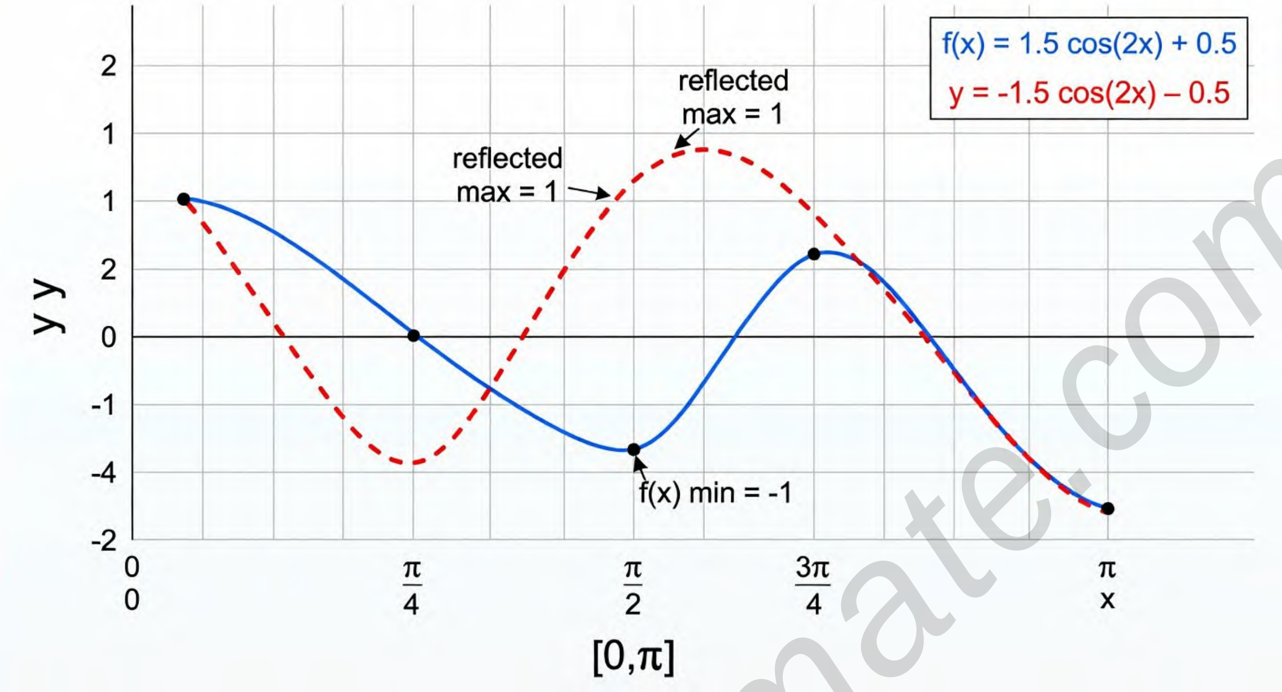
- The transformation that maps $y = f(x)$ onto $y = g(x)$ is a **vertical translation** by the vector $\binom{0}{1}$.

3. Determine the equation of the reflection in the x-axis A **reflection** of a curve $y = f(x)$ in the x-axis is given by the transformation $y = -f(x)$.

- Applying this to the given function:

$$y = -\left(\frac{3}{2}\cos 2x + \frac{1}{2}\right)$$
$$= -\frac{3}{2}\cos 2x - \frac{1}{2}$$

- Comparing this to the required form $y = a\cos 2x + b$, we identify the constants as $a = -\frac{3}{2}$ and $b = -\frac{1}{2}$.



(a) Range: $-1 \leq f(x) \leq 2$
(b) $k = 1$; Translation by the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (or a vertical translation 1 unit upwards)
(c) $y = -\frac{3}{2}\cos 2x - \frac{1}{2}$

9709_11_Summer_2020_Q5

Solution

The problem involves finding the relationship between parameters of a line and a curve based on their intersection properties. We are given the line $y = mx + c$ and the curve $xy = 16$.

1. Tangency Condition

To find the condition for the line to be a **tangent** to the curve, we substitute the expression for y from the line equation into the curve equation:

$$x(mx + c) = 16$$
$$mx^2 + cx - 16 = 0$$

For the line to be tangent to the curve, this quadratic equation must have exactly one real solution. This occurs when the **discriminant** (D) is equal to zero.

- The coefficients are $a = m$, $b = c$, and $c_{\text{const}} = -16$.
- The discriminant is given by $D = b^2 - 4ac_{\text{const}}$.

$$D = c^2 - 4(m)(-16)$$
$$= c^2 + 64m$$

Setting the discriminant to zero for tangency:

$$c^2 + 64m = 0$$
$$64m = -c^2$$
$$m = -\frac{c^2}{64}$$

$$\boxed{m = -\frac{c^2}{64}}$$

2. Intersection at Two Distinct Points

Given $m = -4$, the equation for the intersection points becomes:

$$-4x^2 + cx - 16 = 0$$
$$4x^2 - cx + 16 = 0$$

For the line to intersect the curve at two **distinct points**, the discriminant of this quadratic equation must be strictly greater than zero ($D > 0$).

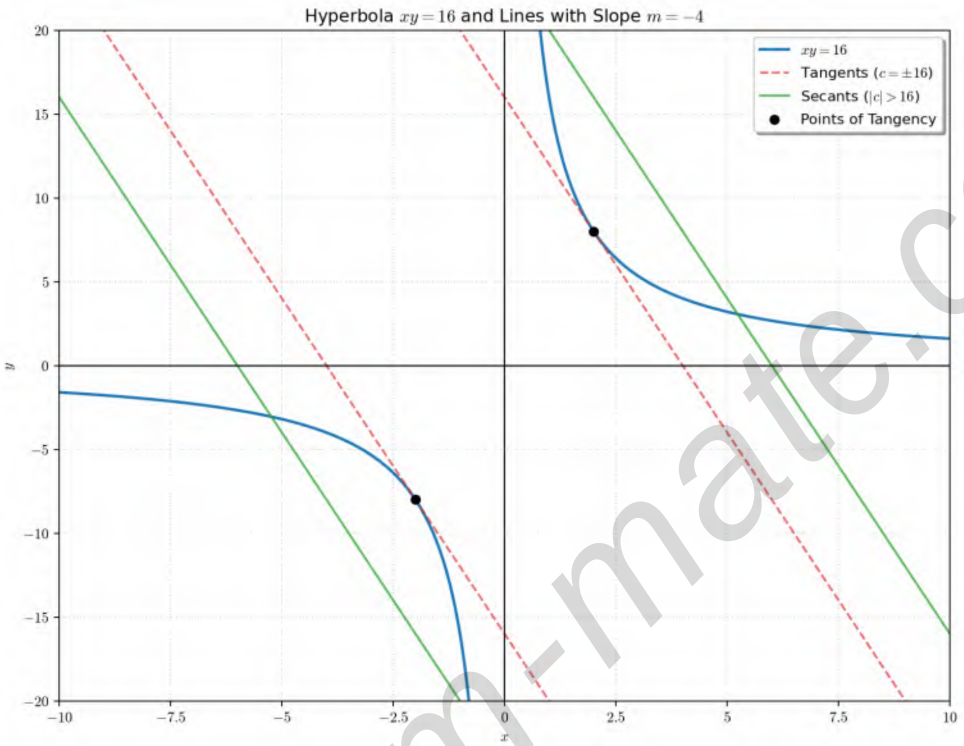
- Here, $a = 4$, $b = -c$, and $c_{\text{const}} = 16$.

$$D = (-c)^2 - 4(4)(16)$$
$$= c^2 - 256$$

We require $D > 0$:

$$c^2 - 256 > 0$$
$$c^2 > 256$$
$$|\, c \,| > \sqrt{256}$$
$$|\, c \,| > 16$$

This inequality is satisfied when $c > 16$ or $c < -16$.



$\boxed{c < -16 \quad \text{or} \quad c > 16}$

9709_11_Summer_2020_Q6

Solution

The problem asks for the values of constants a and b based on given **composite function** and **inverse function** values, and then to find a specific composite expression.

1. Finding the values of a and b

- **Determining b from $gg(2) = 10$:** The function g is defined as $g(x) = 3x + b$. The composite function $gg(2)$ is calculated as $g(g(2))$. First, evaluate $g(2)$:

$$g(2) = 3(2) + b = 6 + b$$

  Next, substitute this into $g(x)$:

$$\begin{aligned} g(g(2)) &= 3(6 + b) + b \\ &= 18 + 3b + b \\ &= 18 + 4b \end{aligned}$$

  Given $gg(2) = 10$:

$$\begin{aligned} 18 + 4b &= 10 \\ 4b &= 10 - 18 \\ 4b &= -8 \\ b &= -2 \end{aligned}$$

- **Determining a from $f^{-1}(2) = 14$:** The function f is defined as $f(x) = \frac{1}{2}x - a$. By the definition of an **inverse function**, if $f^{-1}(2) = 14$, then $f(14) = 2$. Substitute $x = 14$ into the expression for $f(x)$:

$$\begin{aligned} f(14) &= \frac{1}{2}(14) - a \\ 2 &= 7 - a \\ a &= 7 - 2 \\ a &= 5 \end{aligned}$$

Thus, the values are $\boxed{a = 5, b = -2}$.

2. Finding the expression for $gf(x)$

Using the values $a = 5$ and $b = -2$, the functions are:

$$f(x) = \frac{1}{2}x - 5$$

$$g(x) = 3x - 2$$

To find the **composite function** $gf(x)$, we substitute $f(x)$ into $g(x)$:

$$\begin{aligned} gf(x) &= g(f(x)) \\ &= 3\left(\frac{1}{2}x - 5\right) - 2 \\ &= \frac{3}{2}x - 15 - 2 \\ &= \frac{3}{2}x - 17 \end{aligned}$$

The expression is in the form $cx + d$ where $c = \frac{3}{2}$ and $d = -17$.

$$\boxed{gf(x) = \frac{3}{2}x - 17}$$

9709_11_Summer_2020_Q7

Solution

1. Proof of the Trigonometric Identity

To prove the identity $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} \equiv \frac{2}{\cos\theta}$, we start by manipulating the left-hand side (LHS) to find a common denominator.

- **Step 1: Combine the fractions** The common denominator is $\cos\theta(1+\sin\theta)$.

$$\text{LHS} = \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

- **Step 2: Expand the numerator** Expanding $(1+\sin\theta)^2$ gives:

$$\text{LHS} = \frac{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

- **Step 3: Apply the Pythagorean Identity** Using the **Pythagorean identity** $\sin^2\theta + \cos^2\theta = 1$:

$$\begin{aligned}\text{LHS} &= \frac{1 + 2\sin\theta + 1}{\cos\theta(1+\sin\theta)} \\ &= \frac{2 + 2\sin\theta}{\cos\theta(1+\sin\theta)}\end{aligned}$$

- **Step 4: Factor and simplify** Factoring out a 2 from the numerator:

$$\begin{aligned}\text{LHS} &= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} \\ &= \frac{2}{\cos\theta} = \text{RHS}\end{aligned}$$

  The identity is proven.

2. Solving the Equation

We are given the equation:

$$\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{3}{\sin\theta}, \quad \text{for} \quad 0 \le \theta \le 2\pi$$

- **Step 1: Substitute the proven identity** From part (a), the LHS is equal to $\frac{2}{\cos\theta}$. Substituting this into the equation:

$$\frac{2}{\cos\theta} = \frac{3}{\sin\theta}$$

- **Step 2: Rearrange to find $\tan\theta$** Multiplying both sides by $\sin\theta$ and dividing by 2:

$$\begin{aligned}\frac{\sin\theta}{\cos\theta} &= \frac{3}{2} \\ \tan\theta &= 1.5\end{aligned}$$

- **Step 3: Determine the solutions in the interval** $[0, 2\pi]$ Since $\tan\theta$ is positive, θ must lie in the first or third quadrant.
  - **First Quadrant solution:**

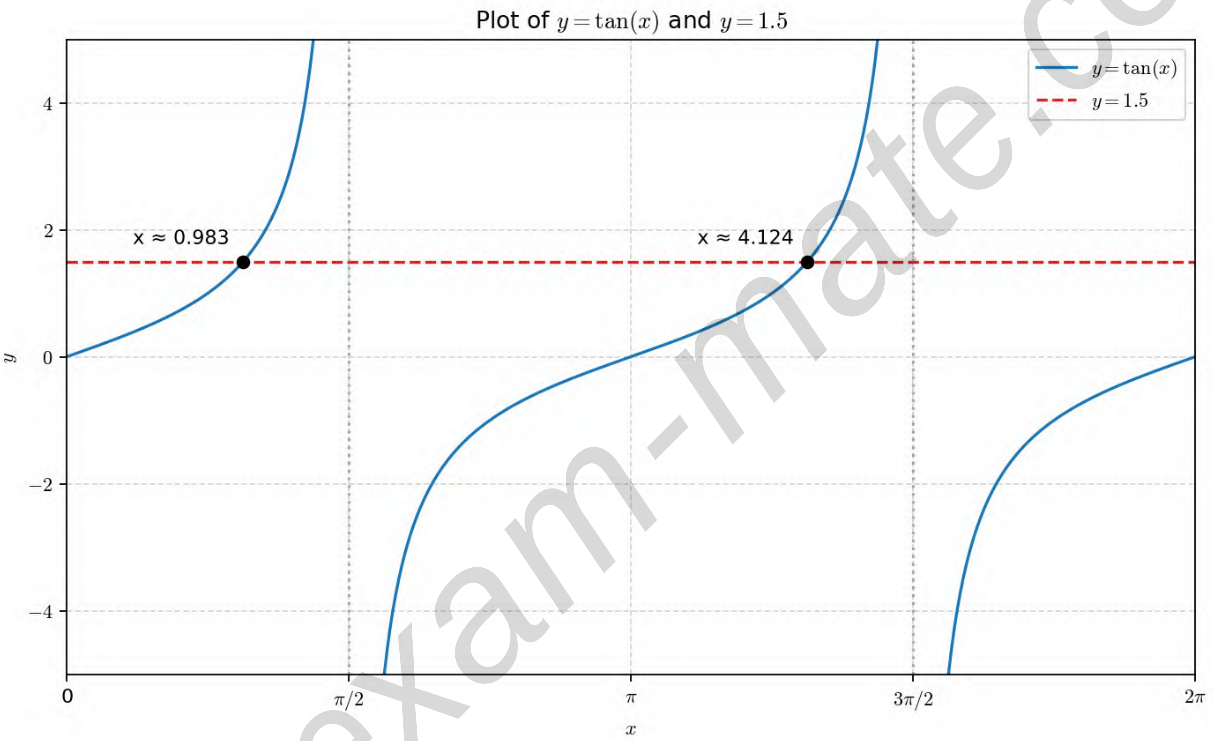
$$\theta_1 = \arctan(1.5) \approx 0.98279...$$

  - **Third Quadrant solution:**

$$\theta_2 = \pi + \arctan(1.5) \approx 3.14159 + 0.98279 \approx 4.12438...$$

- **Step 4: Final values** Rounding to three decimal places:

$$\theta_1 \approx 0.983\,\text{rad}$$
$$\theta_2 \approx 4.124\,\text{rad}$$



$\boxed{\theta \approx 0.983, 4.124}$

9709_11_Summer_2020_Q8

Solution

The problem asks for the perimeter of the shaded region BXC in a semicircle with center O and radius $r = 6\,\text{cm}$. The perimeter consists of three segments: the vertical line segment BX, the horizontal line segment XC, and the circular arc BC.

1. Determine the central angle $\angle AOB$ The length of the **arc** AB is given as $s_{AB} = 15\,\text{cm}$. Using the formula for arc length $s = r\theta$, where θ is in radians:

$$\begin{aligned}\theta_{AOB} &= \frac{s_{AB}}{r} \\ &= \frac{15}{6} \\ &= 2.5\,\text{rad}\end{aligned}$$

2. Determine the central angle $\angle BOC$ Since AC is the diameter of the semicircle, the total angle $\angle AOC$ is $\pi\,\text{rad}$. The angle $\angle BOC$ is:

$$\begin{aligned}\theta_{BOC} &= \pi - \theta_{AOB} \\ &= \pi - 2.5\,\text{rad}\end{aligned}$$

3. Calculate the length of arc BC The length of the arc BC is given by:

$$\begin{aligned}s_{BC} &= r \cdot \theta_{BOC} \\ &= 6(\pi - 2.5) \\ &= 6\pi - 15\,\text{cm}\end{aligned}$$

4. Calculate the lengths of BX and OX In the right-angled triangle $\triangle BOX$, the hypotenuse OB is the radius $r = 6\,\text{cm}$. Using **trigonometry**:

- The vertical segment BX is:

$$\begin{aligned}BX &= OB\sin(\theta_{BOC}) \\ &= 6\sin(\pi - 2.5) \\ &= 6\sin(2.5)\,\text{cm}\end{aligned}$$

- The horizontal segment OX is:

$$\begin{aligned}OX &= OB\cos(\theta_{BOC}) \\ &= 6\cos(\pi - 2.5) \\ &= -6\cos(2.5)\,\text{cm}\end{aligned}$$

(Note: Since $2.5\,\text{rad}$ is in the second quadrant, $\cos(2.5)$ is negative, making OX positive.)

5. Calculate the length of XC The segment XC is the difference between the radius OC and the segment OX:

$$\begin{aligned}XC &= OC - OX \\ &= 6 - (-6\cos(2.5)) \\ &= 6 + 6\cos(2.5)\,\text{cm}\end{aligned}$$

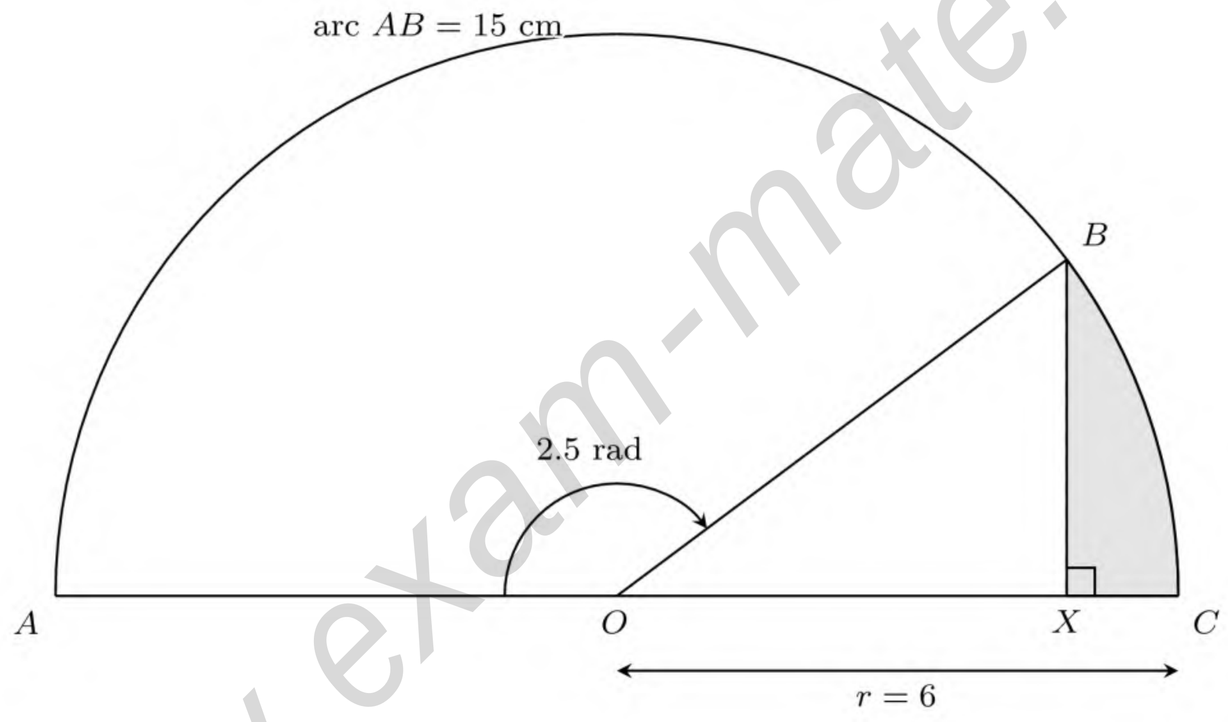
6. Calculate the total perimeter of the shaded region BXC The perimeter P is the sum of BX, XC, and arc BC:

$$\begin{aligned} P &= BX + XC + s_{BC} \\ &= 6\sin(2.5) + (6 + 6\cos(2.5)) + (6\pi - 15) \\ &= 6\sin(2.5) + 6\cos(2.5) + 6\pi - 9 \end{aligned}$$

Using numerical values:

- $6\sin(2.5) \approx 3.5908$
- $6\cos(2.5) \approx -4.8068$
- $6\pi \approx 18.8496$

$$\begin{aligned} P &\approx 3.5908 + 6 - 4.8068 + 18.8496 - 15 \\ &\approx 8.6336\,\text{cm} \end{aligned}$$



The perimeter of the shaded region BXC is approximately $8.63\,\text{cm}$.

$\boxed{8.63\,\text{cm}}$

9709_11_Summer_2020_Q9

Solution

The problem asks for the first and second derivatives of a given cubic function, the coordinates of its stationary points, and the classification of those points.

1. Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

To find the first derivative, we apply the **Power Rule** and the **Chain Rule** to the term $(3-2x)^3$.

- Let $u = 3 - 2x$, then $y = u^3 + 24x$.
- $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} + \frac{d}{dx}(24x)$.
- $\frac{dy}{du} = 3u^2$ and $\frac{du}{dx} = -2$.

$$\begin{aligned}\frac{dy}{dx} &= 3(3-2x)^2 \cdot (-2) + 24 \\ &= -6(3-2x)^2 + 24\end{aligned}$$

To find the second derivative, we differentiate $\frac{dy}{dx}$ with respect to x:

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d}{dx}[-6(3-2x)^2 + 24] \\ &= -6 \cdot 2(3-2x)^1 \cdot (-2) + 0 \\ &= 24(3-2x)\end{aligned}$$

2. Find the coordinates of each of the stationary points

A **stationary point** occurs where the first derivative is zero: $\frac{dy}{dx} = 0$.

$$\begin{aligned}-6(3-2x)^2 + 24 &= 0 \\ -6(3-2x)^2 &= -24 \\ (3-2x)^2 &= 4 \\ 3-2x &= \pm 2\end{aligned}$$

Solving for x:

- Case 1: $3 - 2x = 2 \implies 2x = 1 \implies x = 0.5$
- Case 2: $3 - 2x = -2 \implies 2x = 5 \implies x = 2.5$

Now, find the corresponding y-coordinates using $y = (3-2x)^3 + 24x$:

- For $x = 0.5$: $y = (3 - 2(0.5))^3 + 24(0.5) = 2^3 + 12 = 8 + 12 = 20$.
- For $x = 2.5$: $y = (3 - 2(2.5))^3 + 24(2.5) = (-2)^3 + 60 = -8 + 60 = 52$.

The stationary points are $(0.5, 20)$ and $(2.5, 52)$.

3. Determine the nature of each stationary point

We use the **Second Derivative Test** by substituting the x-values into $\frac{d^2y}{dx^2} = 24(3-2x)$.

- At $x = 0.5$:

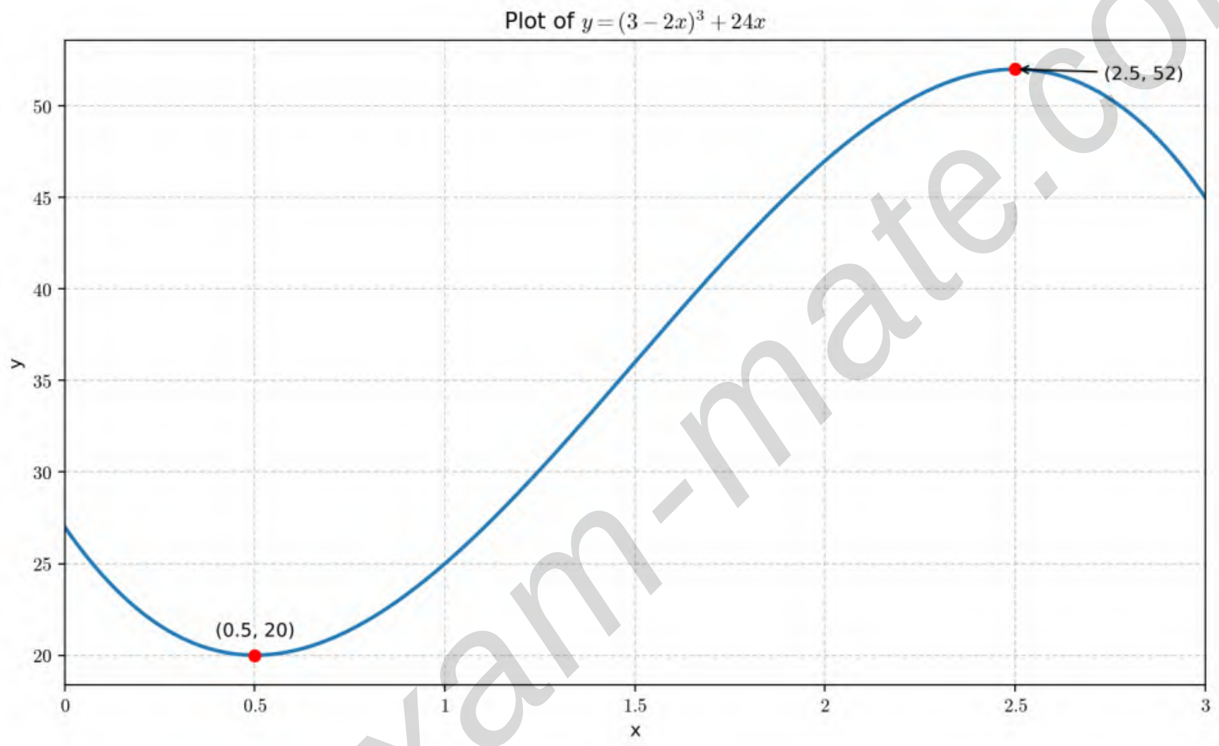
$$\frac{d^2y}{dx^2} = 24(3 - 2(0.5)) = 24(2) = 48$$

Since $\frac{d^2y}{dx^2} > 0$, the point $(0.5, 20)$ is a **local minimum**.

- At $x = 2.5$:

$$\frac{d^2y}{dx^2} = 24(3 - 2(2.5)) = 24(-2) = -48$$

Since $\frac{d^2y}{dx^2} < 0$, the point $(2.5, 52)$ is a **local maximum**.



Final Answers:

(a) $\frac{dy}{dx} = -6(3 - 2x)^2 + 24$, $\frac{d^2y}{dx^2} = 24(3 - 2x)$

(b) $(0.5, 20)$ and $(2.5, 52)$

(c) $(0.5, 20)$ is a local minimum; $(2.5, 52)$ is a local maximum.

9709_11_Summer_2020_Q10

Solution

1. Equation of the circle C with diameter AB

- To find the equation of the circle, we first determine its center $M(h,k)$ and radius r. Since AB is a **diameter**, the center is the **midpoint** of $A(-1,-2)$ and $B(7,4)$.

$$h = \frac{-1+7}{2} = 3$$
$$k = \frac{-2+4}{2} = 1$$

Thus, the center is $M(3,1)$.

- The radius r is the distance from the center $M(3,1)$ to point $B(7,4)$:

$$\begin{aligned} r^2 &= (7-3)^2 + (4-1)^2 \\ &= 4^2 + 3^2 \\ &= 16 + 9 = 25 \end{aligned}$$

The radius is $r = \sqrt{25} = 5$.

- The standard **equation of a circle** is $(x-h)^2 + (y-k)^2 = r^2$. Substituting the values:

$$\boxed{(x-3)^2 + (y-1)^2 = 25}$$

2. Equation of the tangent T to circle C at point B

- The **tangent** to a circle at a point is perpendicular to the radius at that point. We first find the gradient of the radius MB, denoted as m_{MB}:

$$m_{MB} = \frac{4-1}{7-3} = \frac{3}{4}$$

- The gradient of the tangent T, denoted as m_T, must satisfy the **perpendicularity** condition $m_T \cdot m_{MB} = -1$:

$$m_T = -\frac{1}{3/4} = -\frac{4}{3}$$

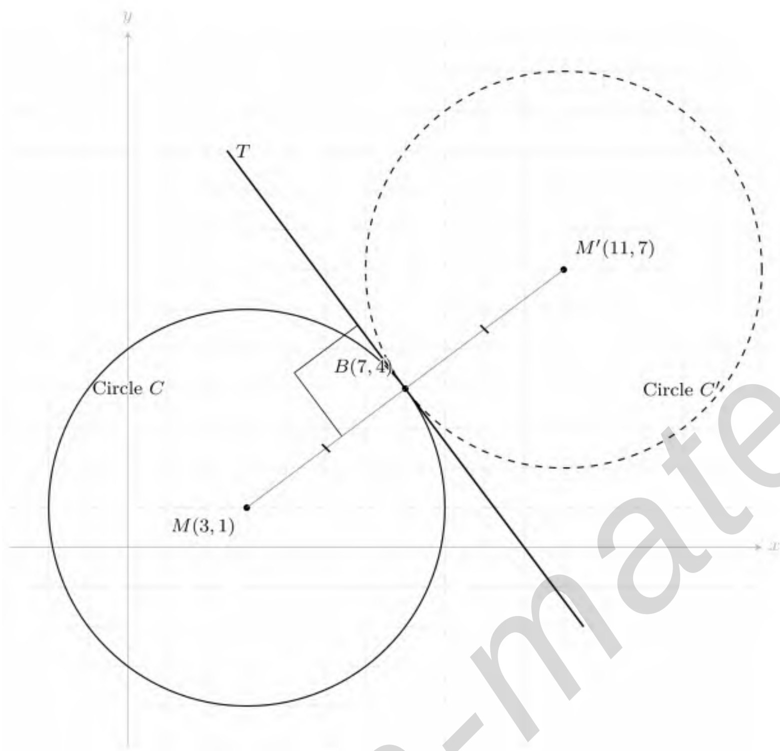
- Using the **point-slope form** with point $B(7,4)$:

$$y - 4 = -\frac{4}{3}(x-7)$$
$$3y - 12 = -4x + 28$$
$$4x + 3y - 40 = 0$$

$$\boxed{4x + 3y = 40}$$

3. Equation of the circle reflected in line T

- When a circle is reflected in a line tangent to it, the radius remains the same, and the center of the new circle is the **reflection** of the original center across the tangent line.
- Since T is tangent to circle C at point B, the point B is the midpoint between the original center $M(3,1)$ and the new center $M'(h',k')$.



$$\frac{3+h'}{2} = 7 \Longrightarrow h' = 14 - 3 = 11$$
$$\frac{1+k'}{2} = 4 \Longrightarrow k' = 8 - 1 = 7$$

The new center is $M'(11,7)$.

- The radius remains $r = 5$, so $r^2 = 25$. The equation of the reflected circle is:

$$\boxed{(x-11)^2 + (y-7)^2 = 25}$$

9709_11_Summer_2020_Q11

Solution

1. Finding the coordinates of A and B

The points A and B are the **intersections** of the curve $y = \frac{8}{x+2}$ and the line $2y + x = 8$. To find these points, we substitute the expression for y from the curve into the equation of the line:

$$2\left(\frac{8}{x+2}\right) + x = 8$$
$$\frac{16}{x+2} + x = 8$$

Multiplying the entire equation by $(x+2)$ to clear the denominator:

$$16 + x(x+2) = 8(x+2)$$
$$16 + x^2 + 2x = 8x + 16$$
$$x^2 - 6x = 0$$
$$x(x-6) = 0$$

This gives two solutions for x: $x_1 = 0$ and $x_2 = 6$.

- For $x = 0$: $y = \frac{8}{0+2} = 4$. Thus, point A is $(0, 4)$.
- For $x = 6$: $y = \frac{8}{6+2} = 1$. Thus, point B is $(6, 1)$.

2. Finding the coordinates of C

The point C lies on the curve, and the **tangent** at C is parallel to the line AB. First, we find the gradient of the line $2y + x = 8$:

$$2y = -x + 8$$
$$y = -\frac{1}{2}x + 4$$

The gradient of the line is $m = -1/2$. The gradient of the curve is given by the **derivative** dy/dx:

$$y = 8(x+2)^{-1}$$
$$\frac{dy}{dx} = -8(x+2)^{-2} = -\frac{8}{(x+2)^2}$$

Setting the derivative equal to the gradient of the line:

$$-\frac{8}{(x+2)^2} = -\frac{1}{2}$$
$$(x+2)^2 = 16$$
$$x + 2 = \pm 4$$

Since C is between A ($x = 0$) and B ($x = 6$), we take the positive root:

$$x + 2 = 4$$
$$x = 2$$

Substituting $x = 2$ into the curve equation: $y = \frac{8}{2+2} = 2$. Thus, point C is $(2, 2)$.

$\boxed{A(0,4), B(6,1), C(2,2)}$

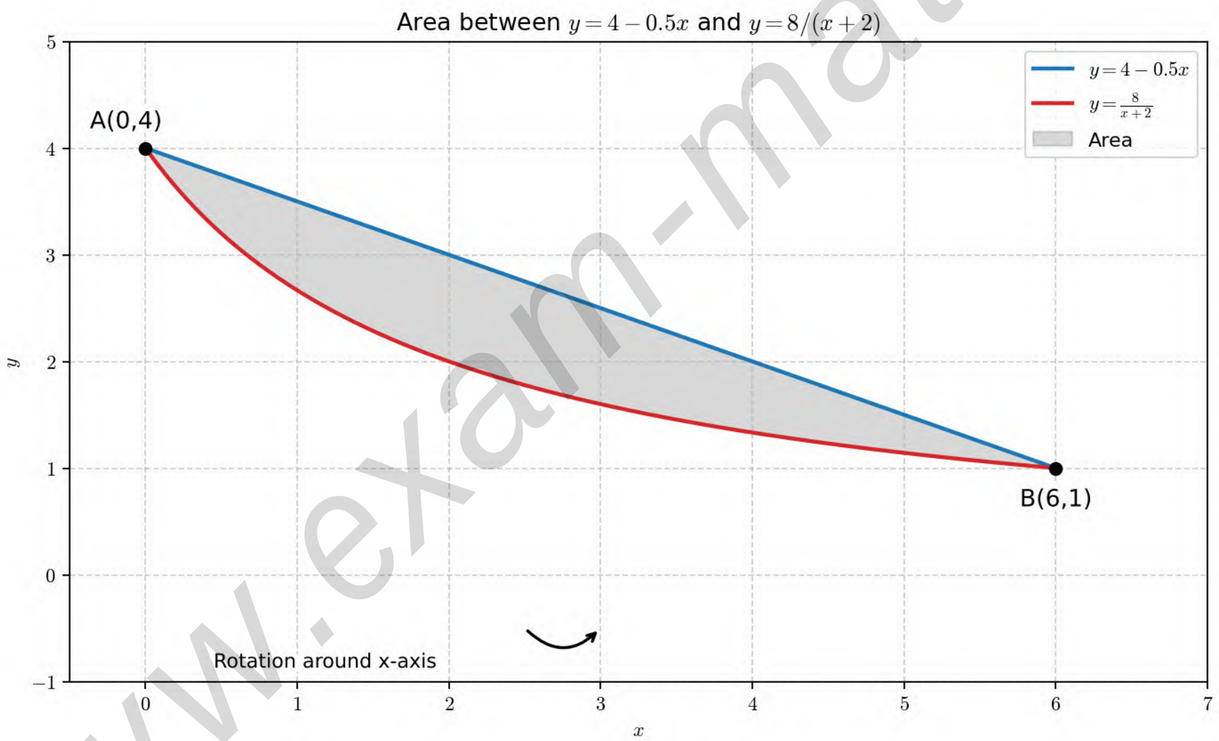
3. Calculating the volume of revolution

The volume V generated by rotating the shaded region about the x-axis is the difference between the volume generated by the line (y_L) and the volume generated by the curve (y_C) from $x = 0$ to $x = 6$. The formula for the **volume of revolution** is $V = \pi \int y^2 dx$.

- For the line: $y_L = 4 - \frac{1}{2}x$
- For the curve: $y_C = \frac{8}{x+2}$

The total volume is:

$$V = \pi \int_0^6 (y_L^2 - y_C^2)\, dx = \pi \int_0^6 \left[\left(4 - \frac{1}{2}x\right)^2 - \left(\frac{8}{x+2}\right)^2 \right] dx$$



Evaluating the integrals separately:

- **Volume under the line (V_L):**

$$V_L = \pi \int_0^6 \left(16 - 4x + \frac{1}{4}x^2\right) dx$$
$$= \pi \left[16x - 2x^2 + \frac{1}{12}x^3\right]_0^6$$
$$= \pi \left(16(6) - 2(36) + \frac{216}{12}\right)$$
$$= \pi(96 - 72 + 18) = 42\pi$$

- **Volume under the curve (V_C):**

$$V_C = \pi \int_0^6 \frac{64}{(x+2)^2} dx$$
$$= 64\pi \int_0^6 (x+2)^{-2} dx$$
$$= 64\pi \left[\frac{(x+2)^{-1}}{-1} \right]_0^6$$
$$= -64\pi \left[\frac{1}{x+2} \right]_0^6$$
$$= -64\pi \left(\frac{1}{8} - \frac{1}{2} \right)$$
$$= -64\pi \left(-\frac{3}{8} \right) = 24\pi$$

- **Total Volume (V):**

$$V = V_L - V_C = 42\pi - 24\pi = 18\pi$$

$$\boxed{18\pi \approx 56.5}$$